interaction ceases when the two modes are sufficiently far from synchronism, even though the guides may still be in close proximity.

The 50-percent (or 3-dB) coupler has more obvious application. No experimental results are available using the tapered coupler design but the computer analysis shows that the improved tolerance to changes in propagation coefficient brings with it no tightening of the tolerance on either coupling coefficient or taper slope.

The present discussion has been limited to passive couplers. The improvement in tolerance would seem to make electrical control, via the electrooptic effect for example, more difficult. However, this can be solved by a suitable circuit rearrangement. For example, switching can be effected by the use of two passive 3-dB couplers and a simple phase shifter [5]. It appears that the present improved design has a direct application here.

Coupling of modes by a periodic perturbation provided either passively or by electrooptic or acoustooptic means is well known. This class of coupling can be "tapered" by simply tapering the pitch of the perturbation, and so avoiding the severe tolerance restriction on the K vector relationship.

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Coupling from Multimode to Single-Mode Linear Waveguides Using Horn-Shaped Structures

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(Invited Paper)

***Abstract*—Coupling from a multimode to a single-mode linear waveguide using horn-shaped structures is investigated. The approximate coupling efficiency is found by numerical solution of coupled-mode equations that apply to the reciprocal problem, i.e., to the problem of propagation in an expanding horn. A coupling efficiency in excess of 90 percent is calculated when coupling is from the principal mode of a sample 50-μm-wide multimode waveguide to a 3-μm-wide single-mode guide ($\lambda = 0.63$ μm). This efficiency results from a uniformly tapered horn whose length is on the order of 2 mm. The length can be decreased by using a shaped coupling region. One such region is found to result in a coupling length of approximately 1.6 mm.**

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INTRODUCTION

THE DESIGN of integrated optical devices finds conflicting requirements. One area where this is true is in the design of modulators. On the one hand, the figure of

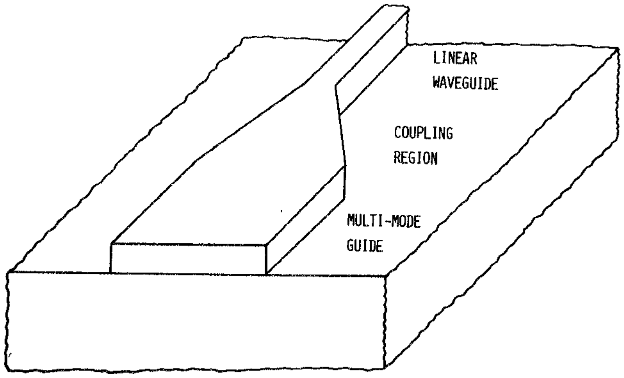

Fig. 1. General structure to be used for coupling light to a single-mode waveguide.

merit (the ratio of drive power at center frequency to bandwidth) improves as the device is reduced to micron or submicron dimensions. On the other hand, insertion losses due to scattering and coupling to fiber transmission lines generally increase with decreased size. The coupling problem can be eased from an experimental and practical point of view if an active waveguide whose cross-sectional area is on the order of square microns can be expanded to widths on the order of tens of microns. This paper is concerned with the effectiveness with which this physical expansion can be accomplished.

Fig. 1 is a representation of the type of coupler considered. The structure consists of a single-mode linear waveguide, a coupling region, and a multimode linear waveguide. We are interested in coupling either to the single-mode linear waveguide or from the single-mode linear waveguide while keeping most of the energy in the lowest order mode. If a wave is launched from the smaller end, then through mode-conversion energy will be coupled to higher order modes as the wave progresses to wider regions of the guide. Typical dimensions considered are waveguide heights between 0.5 and 1.0 wavelengths and coupling regions varying in width from 3 to 50 wavelengths.

The analysis used is a coupled-mode approach where coupling is between the locally normal waveguide modes. We need only investigate the problem of coupling from the smaller to the larger waveguide since the reciprocity theorem allows determination of the coupling efficiency for the other case.

GENERAL FORMULATION

For the structure shown in Fig. 1 the electric and magnetic fields can be decomposed into transverse and longitudinal components. If the longitudinal component is in the z direction then

$$\bar{E} = \bar{E}_t + \bar{E}_z$$
$$\bar{H} = \bar{H}_t + \bar{H}_z. \qquad (1)$$

Then from Maxwell's equations the transverse fields have been shown to satisfy the generalized telegrapher's equations, which for a uniform waveguide and $\exp(-i\omega t)$ time dependence are [1]

$$\frac{\partial \bar{E}_t}{\partial z} = i\omega\mu(\boldsymbol{I} + \boldsymbol{\zeta})\cdot \bar{H}_t \times \hat{z}$$
$$\frac{\partial \bar{H}_t}{\partial z} = i\omega\epsilon(\boldsymbol{I} + \boldsymbol{\zeta})\cdot \hat{z} \times \bar{E}_t \qquad (2)$$

where

$$\boldsymbol{\zeta} = \frac{1}{k^2}\nabla_t\nabla_t$$
$$\nabla_t = \nabla - \hat{z}\frac{\partial}{\partial z}. \qquad (3)$$

Also in keeping with usual notation, μ is the magnetic permeability, ϵ is the dielectric permittivity, $\boldsymbol{I}$ is the identity dyadic, $\hat{z}$ is the unit vector in the z direction, and ω is the angular frequency.

UNIFORM WAVEGUIDE

The transverse fields for a uniform medium can be expressed in terms of an eigenfunction expansion of the form

$$\bar{E}_t = \int_{-\infty}^{\infty} A(\beta,z)\bar{e}(\bar{k}_\rho,x,y)\, d\bar{k}_\rho$$
$$\bar{H}_t = \int_{-\infty}^{\infty} A(\beta,z)\bar{h}(k_\rho,x,y)\, d\bar{k}_\rho \qquad (4)$$

where $\bar{k}_\rho$ is the transverse wavenumber, β is the longitudinal wavenumber, $A(\beta,z)$ is the mode amplitude, and $\bar{e}$ and $\bar{h}$ are eigenfunctions of the region. In the case of dielectric waveguides, $\bar{E}$ and $\bar{H}$ consist of guided modes plus a continuous spectrum; therefore in (4) the integral represents an integral over the continuous spectrum plus a summation over the guided modes [2].

If the waveguide is uniform, then

$$A(\beta,z) = A(\beta)\exp(i\beta z)$$

and $\bar{e}$ and $\bar{h}$ satisfy the differential equations

$$\bar{e}(\bar{k}_\rho,x,y) = \omega\mu/\beta(\boldsymbol{I} + \boldsymbol{\zeta})\cdot\bar{h}(k_\rho,x,y)x\hat{z}$$
$$\bar{h}(k_\rho,x,y) = \omega\epsilon/\beta(\boldsymbol{I} + \boldsymbol{\zeta})\cdot\bar{z} \times \bar{e}(k_\rho,x,y). \qquad (5)$$

Eigenfunctions $\bar{e}$ and $\bar{h}$ have been shown to be orthogonal [3] so we can normalize the fields such that the average power carried by each mode is given in terms of the mode amplitudes, i.e., $P = \frac{1}{2}AA^*$. This normalization is

$$\int \hat{z}\cdot\bar{e}(\bar{k}_\rho,x,y) \times \bar{h}^*(\bar{k}_\rho',x,y)\, da = \delta(\bar{k}_\rho - \bar{k}_\rho')\beta/|\beta|$$
$$\int \hat{z}\cdot\bar{e}_p(\bar{k}_p,x,y) \times \bar{h}_q^*(\bar{k}_q,x,y)\, da = \delta_{pq}\beta_{pq}/|\beta_{pq}| \qquad (6)$$

where $\delta(k_\rho - k_\rho')$ is the Dirac δ-function and δ_{pq} is the Kronecker delta.

NONUNIFORM WAVEGUIDE

When the waveguide is allowed to change its characteristics as a function of the direction of propagation, normal modes can no longer be used. However, $\bar{e}$ and $\bar{h}$ form a complete set for a uniform waveguide so the transverse fields $\bar{E}_t$ and $\bar{H}_t$ can be represented at any cross section by summing all of the locally normal modes. Then β and k_ρ are functions of z and the transverse fields become

$$\bar{E}_t = \int_{-\infty}^{\infty} A(\beta(z),z)\bar{e}(\bar{k}_\rho(z),x,y)\, d\bar{k}_\rho$$

$$\bar{H}_t = \int_{-\infty}^{\infty} A(\beta(z),z)\bar{h}(k_\rho(z),x,y)\, d\bar{k}_\rho. \qquad (7)$$

In the uniform case the modes are orthogonal and can propagate individually; however, the locally normal modes do not individually satisfy the boundary conditions so they cannot propagate alone. In fact, each mode is coupled to all other modes so mode conversion occurs as they propagate in the waveguide.

COUPLED-MODE EQUATIONS

The mode amplitudes of the locally normal modes can be found in terms of coupled differential equations [4]. These equations are:

$$\frac{dA_q}{dz} - i\beta_q A_q = \sum_p C(k_p,k_q)A_p + \int C(k_\rho,k_q)A(k_q)\, dk_\rho$$

$$\frac{dA}{dz} - i\beta A = \int C(k_\rho,k_\rho')A(k_\rho')\, dk_\rho' + \sum_p C(k_\rho,k_p)A_p$$

$$C(k_\rho,k_\rho') = \tfrac{1}{2}[K(k_\rho',k_\rho) - K^*(k_\rho,k_\rho')]$$

$$K(k_\rho',k_\rho) = \int \hat{z}\cdot\bar{e}(k_\rho') \times \frac{\partial\bar{h}}{\partial z}(k_\rho)\, da. \qquad (8)$$

tinuous spectrum within the horn for the indicated direction of propagation. Coupling to the continuous spectrum may be treated in approximate fashion by using a perturbation analysis. Marcuse [5] has examined this coupling for the planar equivalent of the present problem and shown the radiation to be small for shallow tapers. This result may be viewed as being a consequence of the lack of phase match between the continuous spectrum and the guided waves. Phase match is achieved when a mode is at cutoff, but for the forward propagation problem the initial power in higher order modes is zero. Reflected higher order modes propagating toward the narrow portion of the guide will radiate in the vicinity of cutoff, but reflections in the structure can be neglected for small flare angles because the coupling coefficient is small compared with the phase mismatch between forward and backward waves.

Assuming radiation from the guide to be zero decouples the continuous and guided modes in (8). This reduces the reciprocal problem to solutions of the coupled differential equations given by

$$dA_q/dz - i\beta_q A_q = \sum_p C_{pq}A_p. \qquad (9)$$

NORMAL MODES OF A UNIFORM WAVEGUIDE

Before (9) can be solved we need to find the coupling coefficients, which means solving for the normal modes of the waveguide. The normal modes for the linear waveguide of Fig. 2 are given approximately by Unger and Schlosser [6] and Marcatili [7]. The solution basically neglects coupling between TE and TM modes of the planar guide and is useful for guides of large aspect ratio [8]. In regions 6–9 of Fig. 2 we extend the modal solution to include exponential decay in both x and y. With this in mind we define the functions

$$f(x) = \begin{cases} [(-1)^p K_x/(K_x^2 - K_{x5}^2)^{1/2}] \exp[-ik_{x5}(x + t_x/2)], & -\infty < x \le -t_x/2 \\ \cos[k_x(x + t_x/2) - \tan^{-1}K_{x5}/iK_x], & -t_x/2 \le x \le t_x/2 \\ [K_x/(K_x^2 - K_{x3}^2)^{1/2}] \exp[ik_{x3}(x - t_x/2)], & t_y/2 \le x < \infty \end{cases} \qquad (10)$$

$$g(y) = \begin{cases} [(-1)^q \epsilon_1 K_y/\epsilon_4(K_y^2 - K_{y4}^2)^{1/2}] \exp[-ik_{y4}y], & -\infty < y \le 0 \\ \cos(k_y y - \tan^{-1} K_{y4}/ikK_y), & 0 \le y \le t_y \\ [\epsilon_1 K_y/\epsilon_2(K_y^2 - K_{y2}^2)^{1/2}] \exp[ik_{y2}(y - t_y)], & t_y < y < \infty. \end{cases} \qquad (11)$$

The quantities expressed in (8) with arabic subscripts correspond to guided modes and those with Greek or no subscript correspond to the continuous spectrum. Solutions to (8) are difficult to obtain and warrant certain *a priori* approximations. Our basic assumption is that coupling to the continuous spectrum and to reflected waves can be ignored. The justification is that we treat propagation from a narrow to a wide guide and we restrict our consideration to narrow flare angles. In the limits of small or large flare angles there is no coupling to the con-

The modes are labeled E_{pq}^y or E_{pq}^x in accordance with the direction of the transverse component of electric field and are expressed as products of the preceding functions. The transverse-field components of well-confined waves for E_{pq}^y modes are

$$e_y = Nf(x)g(y)$$

$$h_x \simeq -(k_1^2 - k_x^2)/\beta\omega\mu e_y$$

$$N = 2[\beta\omega\mu/(k_1^2 - k_x^2)t_x't_y']^{1/2} \qquad (12)$$

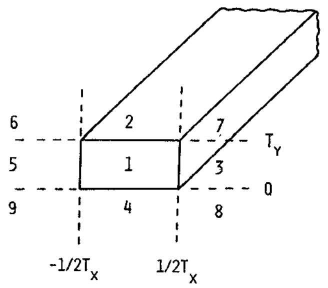

Fig. 2. Linear rectangular waveguide of height t_y and width t_x. The index of refraction for region 1 is greater than all others.

TABLE I
DEFINITION OF TERMS FOR EQUATIONS (10)–(13)

$$t'_x = t_x + \frac{i}{k_{x3}}\left(\frac{K_x}{k_x}\right)\left(\frac{k_xK_x - k_{x3}K_{x3}}{K_x^2 - K_{x3}^2}\right) + \frac{i}{k_{x5}}\left(\frac{K_x}{k_x}\right)\left(\frac{k_xK_x - k_{x5}K_{x5}}{K_x^2 - K_{x5}^2}\right)$$

$$t'_y = t_y + \frac{i}{k_{y2}}\left(\frac{K_y}{k_y}\right)\left(\frac{k_yK_y - k_{y2}K_{y2}}{K_y^2 - K_{y2}^2}\right) + \frac{i}{k_{y4}}\left(\frac{K_y}{k_y}\right)\left(\frac{k_yK_y - k_{y4}K_{y4}}{K_y^2 - K_{y4}^2}\right)$$

$$\beta = (k_1^2 - k_x^2 - k_y^2)^{1/2}$$

$$k_{x(y)i} = (k_i^2 - k_1^2 + k_{x(y)i}^2)^{1/2}$$

Eigenvalue Equations for k_x and k_y:

$$k_xt_x = \tan^{-1}(K_{x3}/iK_x) + \tan^{-1}(K_{x5}/iK_x) + p\pi$$

$$k_yt_y = \tan^{-1}(K_{y2}/iK_y) + \tan^{-1}(K_{y4}/iK_y) + q\pi$$

E_{pq}^y modes: $K_{xi} = k_{xi}$ $K_{yi} = k_{yi}/\epsilon_i$

E_{pq}^x modes: $K_{xi} = k_{xi}/\epsilon_i$ $K_{yi} = k_{yi}$

Note: The subscript i, which refers to the various regions, is dropped for the guide ($i = 1$).

and for the E_{pq}^x modes

$$\begin{aligned} h_y &= Mf(x)g(y) \\ e_x &\simeq (k_1^2 - k_x^2)/\beta\omega\epsilon h_y \\ M &= 2[\beta\omega\epsilon_1/(k_1^2 - k_x^2)t_x't_y']^{1/2}. \end{aligned} \tag{13}$$

M and N are the mode normalizations for unit power. The mode propagation constants β, effective guide thicknesses t_x' and t_y', and other parameters that appear in (10) and (11) are shown in Table I. Locally normal modes are found from (12) and (13) by allowing k_x and β to vary with z.

COUPLING COEFFICIENTS

The coupling coefficients can now be found from the locally normal modes. If the variation of $\bar{h}$ with respect to z in (8) is due only to width variations with respect to t_x then $(\partial\bar{h}/\partial z) = (\partial\bar{h}/\partial t_x)(\partial t_x/\partial z)$. The expression to find the coupling coefficients is

$$K_{pq} = \frac{\partial t_x}{\partial z}\int \hat{z}\cdot\bar{e}_p \times \frac{\partial\bar{h}_q}{\partial t_x}\,da. \tag{14}$$

From (14) and the description of the mode families, coupling coefficients between the E_{pq}^x and E_{pq}^y modes are

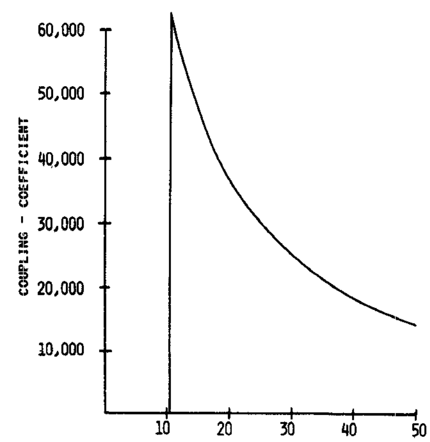

Fig. 3. Coupling coefficient C_{02} as a function of waveguide width t_x for a linear slope such that $\partial t_x/\partial z = 1$. The waveguide is characterized by $n_1 = 1.57$, $n_2 = n_3 = n_5 = 1.0$, $n_4 = 1.53$, $t_y = 0.5$ μm and $\lambda_0 = 0.6328$ μm.

zero so each family can be investigated separately. Also when the coupling region is symmetric with respect to the x axis, coupling occurs only between modes of the same parity. That is, odd modes couple only to odd modes and even modes couple only to even modes. After performing the integration in (14) we find the coupling coefficient between the ith and $(i + 2)$th mode to be zero until the $(i + 2)$th mode begins to propagate; at that point it takes on a peak value and slowly decays. Fig. 3 is a plot of a sample C_{02} assuming $\partial t_x/\partial z = 1$.

Using (12)–(14) one may verify that such coupling coefficients as $C_{0,-p}$ are small under the conditions of interest here and permit a limitation of concern to modes propagating in the forward direction. The coupled differential equations are now solved subject to initial conditions.

COUPLING EFFICIENCY

The reciprocity theorem provides the mechanism for relating the solution of the coupled-mode equations to the efficiency, with which energy is coupled into the narrow guide when a guided beam or wave is incident at the wide end. From the relation [9]

$$\int (\bar{E}^c \times \bar{H} - \bar{E} \times \bar{H}^c)\cdot d\bar{a} = 0 \tag{15}$$

where the superscript c refers to the direct coupling problem, it is simple to show that

$$A_o^c(0) = [\sum A_p^c(z)A_p(z)]/A_o(0) \tag{16}$$

where the summation is over the guided modes at z. In (16) $A_o(0)$ is the guided wave amplitude inside the narrow guide and $A_p(z)$ is the amplitude of the pth mode at some point z along the structure obtained as a solution of the coupled-mode equations. $A_o^c(0)$ and $A_p^c(z)$ are the corresponding coupled quantities.

The power coupled into the single-mode narrow guide is $(1/2)A_o^c(0)A_o^{c*}(0)$ while the amplitude of the coupled-mode incident at z is given in the absence of reflections as the overlap integral

$$A_p^c(z) = \int (\bar{e}_p \cdot \bar{E}^i)\, da \tag{17}$$

where $\bar{E}^i$ is the incident field. The power coupled into the narrow guide as obtained from (16) is

$$P_c = \tfrac{1}{2} \mid \sum A_p^c(z) A_p(z)/A_o(0) \mid^2. \tag{18}$$

In the usual situation we expect the incident field to closely match the lowest order mode so that from (17), $A_o^c(z) \sim 1$, $A_p^c(z) \sim 0$, and from (18)

$$P_c/P_{\text{incident}} \sim \mid A_o(z)/A_o(0) \mid^2 = P_o(z)/P_o(0). \tag{19}$$

The coupling efficiency is thus approximately proportional to the square of the solution of the coupled-mode equations for the lowest order mode. Energy can also be coupled in via higher order modes in the manner indicated by (18).

NUMERICAL CALCULATIONS

The waveguide configuration considered has indices of refractions $n_1 = 1.57$, $n_2 = n_3 = n_5 = 1.0$, and $n_4 = 1.53$. Numerical solutions for linear tapers with a number of different flare angles and waveguide heights are found for these configurations. Power in the lowest modes as a function of length for two typical cases is plotted in Figs. 4 and 5. If $A_o(z) = 1$ and $A^x(z) = 0$, $n \neq 0$, then the lowest mode represents the coupling efficiency.

Insight into the results may be obtained by investigating coupling between two modes with no variations in the propagation constant or coupling coefficients. That is, we wish to solve the two equations

$$dA_0/dz - i\beta_0 A_0 = C_{02} A_2$$
$$dA_2/dz - i\beta_2 A_2 = -C_{02} A_0. \tag{20}$$

Solutions to (20) when mode 0 is launched with unit power is known to be given by [10]

$$P_0(z) = 1 - F \sin^2 \beta_b z$$
$$P_2(z) = F \sin^2 \beta_b z \tag{21}$$

where

$$\beta_b^2 = ((\beta_0 - \beta_2)/2)^2 + C_{02}^2$$
$$F = (C_{02}/\beta_b)^2.$$

In the coupling problem C_{02} and β_i are functions of z so the amplitude of the sine-squared term varies with z in the manner indicated in Figs. 4 and 5.

For the solution in (20) if the phase mismatch $(\beta_0 - \beta_2)/2$ is much greater than C_{02} very little energy can be coupled to the higher mode. Alternatively, if C_{02} is much greater than $(\beta_0 - \beta_2)/2$ nearly all of the energy can be coupled to the higher mode.

With this insight we return to the coupling problem and investigate $(\beta_0 - \beta_2)/2$ and C_{02}. Fig. 6 is a plot $(\beta_0 - \beta_2)/2$ and C_{02} is a function of distance along the coupler. C_{02} is shown for a linear taper with a flare angle of 2°. When a new mode begins to propagate in the coupler the coupling coefficient is peaked; however, the difference between β_0 and β_2 is also peaked so that, at least for small angles,

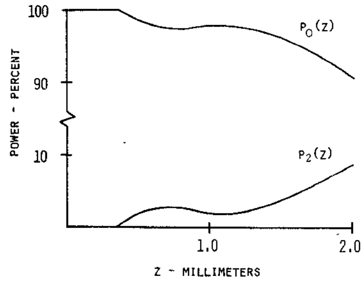

Fig. 4. Power distribution in the two lowest modes for an input of unit power. Width at the input end is 3 μm. The waveguide is characterized by $n_1 = 1.57$, $n_2 = n_3 = n_5 = 1.0$, $n_4 = 1.53$, $t_y = 0.5$ μm, and $\lambda_0 = 0.6328$ μm. The width variation is $t_x = t_0 + 2z \tan \theta/2$ where $t_c = 3$ μm and $\theta = 1.3°$.

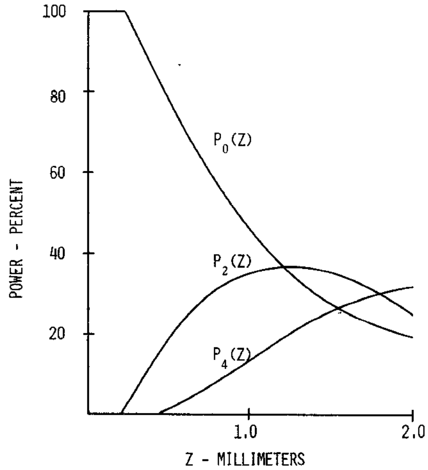

Fig. 5. Same as figure with $\theta = 3°$.

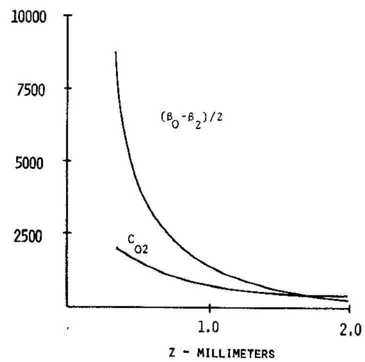

Fig. 6. Difference in β for the two lowest order modes and coupling coefficient between them as a function of distance along the coupler. The structure is the same as in Fig. 5.

coupling remains small close to the cutoff thickness of the higher order mode.

Coupling efficiency can be improved by shaping the walls of the coupler. An improved design should, of course, have zero slope at the junction of the coupler and the linear waveguide to reduce losses due to radiation. Near the cutoff width of the higher order mode the slope can be increased with respect to the linear case and still maintain low energy conversion. Finally, the slope should decrease as the width of the coupler increases and phase match is approached.

A coupler with a width variation such that $t_x = t_o + A(1 - \exp[-z/l]) + 2z \tan \theta/2$ allows for the adjust-

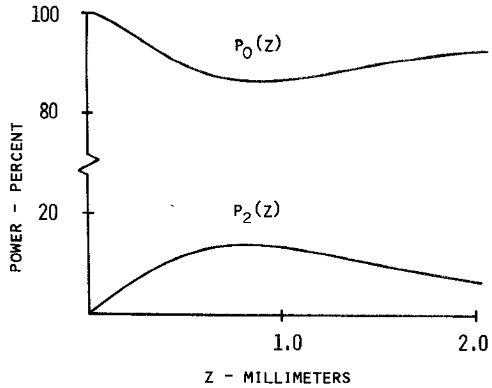

Fig. 7. Same as Fig. 5 but having a width variation $t_x = t_0 + A(1 - \exp[-z/l]) + 2z \tan \theta/2$ for $t_0 = 3$ μm, $A = 51$ μm, $l = 1$ mm, and $\theta = 0.2°$.

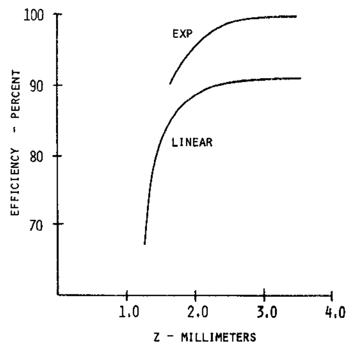

Fig. 8. Coupling efficiency at a horn width of 50 μm as a function of coupler length for the linear taper ($t_x = t_0 + 2z \tan \phi/2$) and the exponential taper ($t_x = t_0 + A(1 - \exp[-z/l]) + 2z \tan \theta/2$) described in Figs. 5 and 7.

ment of the coupling coefficient. In particular, the coupling coefficient can be adjusted as described in the preceding. Fig. 7 is a typical plot of the power in the two lowest modes for a coupler of this design.

By comparison Fig. 8 shows the coupling efficiency for a linear taper of various angles and the improved taper again for various angles at a width of 50 μm. The waveguide used in this example has the following indices: $n_1 = 1.57$, $n_2 = n_3 = n_5 = 1.0$, and $n_4 = 1.53$, waveguide height is 0.5 μm and taper goes from 3.0 to 50 μm in width. The free-space wavelength λ_0 equals 0.6328 μm.

CONCLUSIONS

Coupling from a multimode linear waveguide to a single-mode linear waveguide can be investigated by using coupled-mode theory. The coupling efficiency is found by solving what is called the reciprocal problem then applying the reciprocity theorem. When coupling efficiencies greater than 90 percent are desired, calculations for the sample structure considered here show that the coupling region must be on the order of 2000 wavelengths for a linear taper. This distance is appreciably reduced by shaping the coupling region.

Ostrowsky *et al.* [12] have recently reported fabrication of horn-shaped couplers using electron resist for the waveguides and a computer-controlled scanning-electron microscope to produce the shape. Quantitative results of coupling efficiency were not reported.

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